Abstract: Badiou’s essay *Mark and Lack: On Zero* was published in the last volume of *Cahiers pour l’Analyse* (Cahiers) in 1967 shortly before the May 1968 events in France. In *Mark and Lack* Badiou provides a sustained critique of two essays published in the earlier volumes of the same journal by Jacques Alain Miller called *Actions of Structure* and *Suture* (see the bibliography). The latter two essays provide an effort to bring together the categories of structure and the subject in a theory that borrows from psychoanalysis (Lacan) and mathematical logic (Frege), dubbed by Miller the ‘logic of signifier’. In *Mark and Lack* Badiou criticized this theory in its metaphysical assumptions as well as epistemological approach. In this essay, we read Badiou’s *Mark and Lack* closely and reconstruct its major arguments. But more importantly this essay attempts to show that *Mark and Lack* should be read as the first chapter of a larger project which culminated in Badiou’s magnum opus *Being and Event* by establishing the foundational concepts of discipline and interiority, and by showing that *Cahiers* and psychoanalysis commitment to science is not thorough enough. The introduction of indeterminism and non-identity to the science perceived as the realm of self-identity by psychoanalysis is abrupt and ideological. It is exactly by deepening the commitment to scientific formalism and determination that Badiou finally opened the path to indeterminism and non-self-identity, of the entire situation of being. Non-identity is the law of being not of the subject.

Keywords: Epistemology, Logic, Computability, Epistemological Rupture, Subject and Structure, Suture, Discourse, Discipline, Interiority

Jacques-Alain Miller delivered the lecture named *Suture* as an intervention at Lacan’s seminar *Critical Problems for Psychoanalysis* on Feb 24th 1965, a few days after his 21st birthday. Suture is a word picked from the ordinary language and is used by Miller to apply to a very specific field in the Lacanian psychoanalysis – the logic of the signifier. Central to the efforts of Miller was Gottlob Frege’s conception and generation of natural numbers. Frege believed that numbers are logical constructs and are generated based on pure thought: what has been referred to as Frege’s logicism. The construction of number in Frege’s system was a purely logical task, but according to Miller the general field of logic used by Frege in order to generate the concept of cardinal number and the concept and generation of natural numbers, is itself rooted in a more ‘primordial’ logic, the logic of signifier, which in Miller’s analysis thematizes the Frege’s generation of cardinals. Central to this logic is the notion of the subject as a role or position within the structure that while it holds the structure...
together, it captures the relation of the system to something that does not exist but yet ‘determines’ the system. Subject is thus a conceptual apparatus that performs two crucial roles in any structure:

1. It inscribes what does not exist as something that is registered within the structure.
2. It allows for what does not exist to cause the structure to exist and to expand.

In short, the claim made by Miller is that structured ensembles (and the sequence of cardinals is an example of such ensembles) are all built around a subjective core – it is this subjective core that is the ‘essence’ of such structures, or no structure exists without a repressed subjective core. In this sense subject is a meta-logical or even ontological apparatus, and by no means refers to anything experiential or substantial or objective, which usually denotes human individuals. There are only two discourses that do not repress this subjective core: psychoanalysis and Marxism. Miller calls these the discourses of overdetermination. The larger programme, of which Suture is a piece, is to provide a general theory that unifies these or any other discourses of overdetermination. Miller calls this general theory the unified theory of discourse.¹

Badiou’s Mark and Lack is a sustained critique of Miller’s larger programme in general and of Suture as its demonstrative piece in particular. Right out of the gate, in the very first paragraph of this seminal essay Badiou lays his cards on the table, so to speak. It is in the spirit of the epistemological rupture that he speaks, the rupture that breaks epistemology away from ideology (and common sense alike, one might add) as it evacuates from science any notion of Truth, and replaces it with “a mechanism of production”².

What does this rupture say about logic? The question here is what does Badiou have in mind?

Badiou says that there is an ideological representation of logic too in which there exists a presupposition of the positing of Truth rather than the construction of an object. In this representation, which Frege is in part responsible for by abruptly likening “a proposition to a proper name whose reference, or denotation, is the True or the False”, “logic incessantly coordinates as many linked inscriptions as necessary in order for it to pass from one invariable name-of-the-True to another”³.

Classical logic focuses on the forms and categories of statements in order to sort through valid and invalid conclusions. It involves the study of the truth-involving relations between sentences; it is interested in giving a general answer to the question: when does the truth of one set of sentences guarantee the truth of some other sentence, or what does it take for one sentence to follow from some others? Doing this involves giving some kind of analysis of sentences into their parts, since whether one sentence follows from another is typically a matter of relations between parts of those sentences. The prime example of such analyses is the Aristotelian subject/predicate logic, which breaks down the sentences into subject part and predicate part, and determines that the most general distinctions between classes of sentences are distinctions between modes of predication, and the fact that we can explain valid inference by suitably categorizing the subject, and the predicate, of the sentences involved in the relevant argument.

In contrast modern logic, i.e. the logic in the context Badiou is talking about, which is the logic usually assigned to Frege, is worried about the truth-values of statements. In another word Frege is worried about the instances of a statement, and if two statements have the same instances then they are equal statements. Therefore, all statements for Frege are (complex) denoting terms: they are terms that denote truth-values. This is an important difference between classical and modern logic. In classical logic, the quantifiers did not play a significant role. With Frege on the other hand we have the transcription of the old statements of categorical logic in a language employing variables, quantifiers and truth-functions.

The modern logic, mostly with Frege, invented modern quantification theory, presented the first complete axiomatization of propositional and first-order ‘predicate’ logic (the latter of which Frege invented outright).

This is an important clarification for Badiou. The revolution inaugurated by Frege in logic had the intent to reduce mathematics to logic and logic to a conceptual construction of truth functions. This not only makes logic a system of conceptual construction foreign to the real, it also makes mathematical objects and the mathematical theory an exercise in tautology, also completely foreign to the real – concept comes first and number thereafter. We know from later Badiou that this direction is in a direct opposition to what he has in mind about the role mathematics plays, as ontology – it is not only not tautological, it is the science of being qua presentation. What is important here is that even at this stage of his intellectual career Badiou notices that the direction of equating logic, and with it the entire mathematics, to a complete conceptual construction is a sort of metaphysics in disguise, one that is at the service of producing

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¹ Miller 2012-A, p.71
² Badiou 2012-A, p.159
³ Badiou 2012-A, p.159
ideology of science and not the science itself.

It is the latter interpretation of logic formalized by Frege that is at the center of theorization in Miller (and Lacan). This interpretation starts from the conceptual formalization of zero based on the Leibniz principle of identity, and propagated through the number system, such that each number is the recounting of the same non-identity principle. Miller has obviously used this conceptualization of zero and number in Frege's system as the basis of his concept of suture, central to the logic of the signifier. The non-identity is of course the subject, whose subjective implication of its non-reflective part in the structure is masked by its reflective part, and yet bound through a causal connection, the non-reflective and reflective parts remain inseparable. The reflective part reduplicates the reality in the imaginary and the non-reflective registers itself as an absence, a lack, but the two parts feed off each other in a repetitive entanglement through which the more lack presses on the more imaginary will prevail, which in turn intensifies the force of the lack. But in this relationship, the imaginary is the reduplication, that is, recounting of the lack every time. This is how, as remarked by Badiou, for Miller True is another name for the lack.

The nominal movement, the repetitive compulsion that, in the chain of propositions, unravels our disbelief in the True’s common patronym, marks nothing but the lack over which this movement glides without resistance or success.5

To this logic, containing the two folded process of reduction to lack (True) and the reduction of the latter to non-identity, Badiou wants to posit a different logic: the logic of stratification, in order to show the true closure and foreclosure of science (and more particularly mathematics) such that within it, it does not lack anything that it doesn’t produce elsewhere, and such that he could finally show that: “The logic of the Signifier is a metaphysics: a representation of representation, an intra-ideological process and progression.”6

What is at stake in our view however is not just whether two views of science, one more or less attributed to Althusser and one being worked out by Miller and other Cahiers’ editors, can converge or they indeed diverge. We think the question for Badiou is really whether the logic of the signifier can commit to the epistemological rupture or not. Badiou, as we will witness, is committed to the original Bachelardian project, in which the objective process of science lacks nothing it cannot produce within itself. The issue however is that in Badiou’s mind, Miller et al have attempted to bypass this notion of scientificity by succumbing to Frege, whose method allows them to identify a repressed element in science: the non-self-identical element that is repressed and then sutured to the entire process. This allows Miller et al to import a primitive into the foundational theory: the subject. It seems to me then that the real problem for Badiou in accepting the theory of the signifier hovers exactly around the same point: the point of subject within the structure. For Lacan and Miller the entire process of language is marred by an imaginary process. This process is necessary for the dynamics that is embedded in the speech. For this dynamics to hold ground it is necessary to assume in the clinical setting the role of a reflective element: in that sense for the clinical setting it is a mandatory assumption. Miller’s ambition however is to use the same reflective element in a much wider stage, that is for the entire science. He is generalizing something that is operative to a specific field of human sciences, beyond its applicability.

The thesis we are defending here aims only at delineating the impossibility of a logic of the Signifier that would envelop the scientific order and in which the erasure of the epistemological break would be articulated.7

Therefore, Badiou’s project in Mark and Lack contains three components. First, he wants to show in mathematics, as in the rest of science, which the epistemological rupture demarcates, there is no lack, and nothing within it, including its progression, is motivated or dependent on the functioning of a lack. Second, any deliberation on the foundation of mathematics and science should first consider requirements from within those fields, and not for example from psychoanalysis or historical materialism; it is up to mathematics to define what is required for the foundation of mathematics, and if the lack does not appear anywhere in the theory we should take that hint very seriously as the sign that its foundation does not need a theory pertaining to that notion. And third, as a conclusion, he wants to show that there is an inversion underlying the way in which the theory of the signifier is formulated. The theory of the signifier, which pertains to psychoanalysis is turning the requirement specific to a particular discourse and extends it to the rest of the discipline of science, which to

4 See Miller 2012-A for the general logic of the signifier and the role of the subject and Miller 2012-B for Miller’s appropriation of Frege’s construction to found the logic of signifier.
5 Badiou 2012-A, p.159
6 Badiou 2012-A, p.159
7 Badiou 2012-A, p.160
Badiou is a clear sign of an ideological recapture, “in which every science comes to mime its own reflection”.8

In what follows we will follow Badiou’s argument closely to show each of these points that may be scattered through his remarkably concise and dense essay.

The first thing to note is Badiou’s definition of the theory of logic:

> The theory of logic pertains to the modes of production of a division in linear writing or inscription.9

According to this short concise and preliminary definition logic comprises two main components:

1. Whether for a certain problem there exists an algorithm that can provide the answers.
2. Whether the algorithm is complete, that is for some certain given conditions (e.g. inputs) the algorithm is deterministic and it halts.

In computability theory, there are countless such algorithms. The very existence of these algorithms should be a counter example for the generality of the logic of signifier and the structural dynamics that it intends.

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8 Badiou 2012-A, p.159
9 Badiou 2012-A, p.159

Tom Eyers remarks: “Logic is rendered by Badiou here as a self-constituting, self-perpetuating ‘machine,’ impervious to the vicissitudes of the subject or the signifier.” (Eyers 2013, p.84). This remark does not match Badiou’s project and is more aligned with the constructivist project. Logical machine is not a determinate process. The whole point of the Turing machine is the demonstration of the fact that the determinate algorithmic process may have indeterminate results.

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11 Although it is not clear whether this matters to Badiou and the algorithm he presents whether $S$ is finite or infinite, but it does to the algorithmic behavior of the machine. In other words, it is important to know whether the next operation starts when the first operation halts or not. With the current specification, there is no requirement for such dependency in the function of the two operations: the second operation can start as soon as the first operation reaches an output. This in software design is called trickle-feed.

12 Badiou 2012-A, p.162
The algorithmic nature of the formation operation is guaranteed to result in, as said before, a dichotomy, one of which is the set of well-formed expressions, which Badiou names $E$.

The third operation, derivation, uses the well-formed expressions from set $E$ and determines whether the expression is derivable (it is a thesis in the system) or it is non-derivable (it is a non-thesis in the system). If $e$ is an expression in $E$ and $e$ is a thesis then $e$ belongs to the set of theses $T$ and if $e$ is a non-thesis it belongs to the set of non-theses $NT$.

Furthermore, there is an operator $\neg$ (negation) such that:

$$e \in T \iff \neg e \in NT,$$

and vice versa,

$$e \in NT \iff \neg e \in T.$$  

So, if we have two expressions $e_1$ and $e_2$ both belonging to $E$, and $e_1 \in T$ and $e_2 \in NT$ we can also write $\neg e_1 \in T$ and $\neg e_2 \in NT$.

If the above were true then the derivation operation like the operation before it would have created a perfect dichotomy: $T$ and $NT$, and the perfect dichotomy would have been based on a certain relation that existed between each expression and its negation, such that if one belonged to one set the other belonged to the other set. That is, it would not be possible to have an expression and its negation belonging to the same set. If, as Badiou presents, we use the symbol ‘...’ to denote the relation between an expression and its negation, i.e. ‘$e ... \neg e$’ means the relation between $e$ and $\neg e$, then, following Badiou’s lead, we can say that the mechanism of derivation, in case of a perfect dichotomy, cuts right through the middle of all such relationships according to which each expression and its negation belong to opposite sides of the perfect dichotomy:

**Mechanism of derivation**

$$e \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \neg e$$

If a perfect symmetry such as this existed David Hilbert’s dream of having an effective procedure (an algorithm) capable of proving all truths about axiomatic systems were possible.

Hilbert at the great mathematical congress held in Paris in 1900 posed ten problems to the world of mathematics. In 1928, he rearticulated them in three major categories of problems, which Stephen Hawking summarizes them as follows:\(^{13}\)

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1. To prove that all true mathematical statements could be proven, that is, the completeness of mathematics.
2. To prove that only true mathematical statements could be proven, that is, the consistency of mathematics.
3. To prove the decidability of mathematics, that is, the existence of a decision procedure to decide the truth or falsity of any given mathematical decision.

Gödel’s incompleteness proof in 1931 dashed Hilbert’s hope, or at least part of his hope. Gödel’s incompleteness theorem disproves the first of these challenges.\(^{14}\) He proved that in a consistent system it is possible to have expressions such that neither it nor its negation is provable, that is $e \in NT$ and $\neg e \in NT$ at the same time. Such an expression in Gödel’s term is undecidable. The undecidability of a well-formed expression however does not disturb the fact that the derivation mechanism cuts $E$ into a dichotomy, because it still does. The issue is not that we do not have a dichotomy, because we still do even after Gödel. The issue is that the relation between the two parts of this dichotomy, $T$ and $NT$, is no longer a perfect symmetrical relationship such that when an expression belongs to one set its negation always belongs to the other set, because sometimes an expression and its negation are both un-provable. Therefore, the meaning of incompleteness is this: one of the undecidable expressions ($e$ or $\neg e$) must be a thesis, but in a consistent system we cannot prove either of them, so therefore $T$ is not a complete set of all theses (because we cannot derive some of them).

The fact that Gödel proved the existence of undecidable expressions does not speak to the undecidability of the systems, which corresponds to Hilbert’s third problem. The third problem, the decidability of a system, exists even after Gödel. However, after Gödel, it is no longer possible to prove the truth of all expressions, we have to suffice with determining whether they are derivable or non-derivable (while may still be true).

Perhaps, in passing, it will be useful to provide some remarks regarding the third problem whose aim is not the ability to derive (i.e. to prove or disprove) an individual statement, but to come up with a procedure able to determine any statement in the system (whether it is derivable or not). After Alan Turning we now call this the ‘computability problem’, i.e. a

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13 Hawking 2007, p. 278
14 The second incompleteness theorem, an extension of the first, shows that the system cannot demonstrate its own consistency. (Wikipedia) The second theorem disproves the second problem.
problem that can be solved using a Turing machine, and in computer science, we express this problem as the ability to articulate the solution through an algorithm that a computer can execute. If all axiomatic systems were decidable, what Hilbert had hoped for, then we could say that for every system there could be finitely many algorithms that can decide the status of every correctly formed statement in that system, and determine whether they are true, false or undecidable.

First Alonzo Church and then, almost simultaneously and independently, Alan Turing disproved Hilbert’s decidability problem (using lambda calculus in the case of Church and in the case of Turing using a completely novel method, which we call the Turing machine today).

The intriguing way in which Turing solved this problem gave rise to another problem, which is almost as important as the three problems by Hilbert.

4. Given the existence of an algorithm to solve a problem, is the solution practical?

The informal term ‘practical’, used above, means the existence of an algorithm solving the task that runs in polynomial time on a Turing machine such that the time to complete the task varies as a polynomial function on the size of the input to the algorithm (as opposed to, say, exponential time). Computer scientists call this the complexity problem, based on which they divide questions into two classes: the general class of questions for which some algorithm can provide an answer in polynomial time. They call this ‘class $P$’ or just $P$. In contrast, for some questions, there is no known practical way to find an answer, but if one is provided with information showing what the answer is, it is practical to verify the answer. The class of questions for which an answer cannot be found but can be verified in polynomial time is called $NP$, which stands for ‘non-deterministic polynomial time’. It is important to note that both $P$ and $NP$ classes of problems are solvable problems, i.e. there exist algorithms that can find the answer to their questions, but in the case of $NP$ this answer cannot be given in any practical way.

Despite this categorization it was not possible, before 1971, to prove that a problem is $NP$. In order to prove that a problem is $NP$ we must prove that there is no algorithm that can find answers in polynomial time. In 1971 Stephen Cook found the first $NP$ problem. He proved that the Boolean satisfiability problem is a $NP$ problem. This result is now known as the Cook-Levin theorem. Using this theorem, it was now possible to show certain problems are at least as hard as the Boolean satisfiability problem, and therefore they must also be $NP$. This lead to a new subset of $NP$ problems we now call $NP$-complete problems. $NP$-complete problems are a set of problems to each of which any other $NP$ problem can be reduced in polynomial time, and whose solution may still be verified in polynomial time. That is, any $NP$ problem can be transformed into any of the $NP$-complete problems. Informally, an $NP$-complete problem is an $NP$ problem that is at least as ‘tough’ as any other problem in $NP$.

A number of important and useful problems are proven to be $NP$-complete. For example, the prime factorization problem is a $NP$-complete problem, something that mathematician John Nash hinted at in 1955. Prime factorization is the basis for encryption because when the key is known, its verification is $P$ but when the key is unknown the answer to the algorithm runs in exponential (non-polynomial) time, relative to the length of the key.

The reason this little excursion may be useful is that it accentuates a stark contrast of ultimate importance to Badiou as well as to this project. The early to the mid twentieth century discoveries regarding what is provable, what is computable and what is complex, Gödel, Church and Cook results respectively, are limitations, or better said consequences of axiomatic thinking. They are not signs of a repressed lack in science – it was not the case that scientists are re-experiencing an impossibility that as a traumatic core in science keeps repeating itself – but more so the signs of the affirmative power of science: the fact that science continues to think beyond its determination set out by a particular discourse about the science – which in this case is Hilbert’s program – what the generation of French philosophers at the time, and in particular the Althusser’s milieu, was referring to as the epistemological rupture. One of the central theses of the present project is that the culmination of this ‘rupture’ is what informs and underlies Badiou’s mature work: the excess of being over language, captured by the axiom of actual infinities. This of course was not yet present at the time Badiou was writing his audacious rejoinder to Miller, but from the way Badiou is troubled by Frege’s and Miller’s appropriation of logical laws in order to ideologically re-appropriate certain metaphysical imports, it is clear how Badiou is on the path to discover what will eventually informs his entire project.

What are these metaphysical imports?

The law of self-identity belongs to symbols, which Badiou calls marks, not objects. Identity of marks is an intra symbolic law and has no import
to the realm of physical objects. When we say a symbol \( x \) is identical to \( x \) wherever we write \( x \), we are simply stating a self-evident fact, a fact that its negation is unthinkable. That is, the lack of self-identity of symbols will not find its own symbol within the set of symbols; there is a lack of mark for such an absurdity. The set of marks is foreclosed to such a lack.

Badiou makes the latter point clear through devising a function to represent the equal sign: instead of writing \( x = y \), using this function we write \( I(x, y) \). In that case we have always the following two formulae:

\[
I(x, y), \text{ every variable is equal to itself}
\]

\[
I(x, y) \iff [A(x) \iff A(y)]
\]

The latter formula, written in first order logic, means that for any given function \( A \) if the value of \( A \) is the same for \( x \) and \( y \), it is because \( x \) and \( y \) are equal and vice versa.

What about \( \neg I(x, x) \), a completely permissible or well-formed expression in the system? Can it not be the mark of non-self-identity that we are seeking? Absolutely not! The expression \( \neg I(x, x) \), the formula for self-inequality, is permissible solely on the basis that the first \( x \) in the function is the same as the second \( x \) in the function: the same mark written in two different places are not two different things, which is the meaning of the self-identity of the marks.

The production of the logical concepts of equality and self-inequality presupposes the foreclosure of what is scripturally non-self-identical. The lack of the equal is built upon the absolute absence of the non-identical.

Here one can see the power of stratification that logic is capable of applying to itself. The production of \( I(x, x) \) and \( \neg I(x, x) \) are the outcome of the function of concatenation we saw above. The function of formation puts both of these expressions in \( E \), since they are both well formed. The function of derivation is the one that puts them into two different categories of expressions \( T \) and \( NT \). If the set of true expressions in a logic is the outcome of the last operation (derivation), then in that set there is no presence of \( \neg I(x, x) \). The identity of marks or graphemes is the law of the first operation (concatenation), whereas the expression of self-equality as the truth is the outcome of the third operation (derivation).

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16 This is an important distinction from the point of view of Badiou’s later development. The operation of count-as-one is an operation that belongs to situations not being qua being as inconsistent multiplicity.

17 Badiou 2012-A, p.167

18 There is a powerful consequence of this method of stratification, which will become important in Badiou’s later development. In set theory, there is a clear distinction between construction of sets extensionally (by picking elements from other existing sets) or intensionally (by declaring a formula that defines a set). Russell’s paradox shows the latter definition of sets is inconsistent – which eventually resulted in having an axiom in set theory called the axiom of Separation. One of the consequences of the Russell’s paradox is that there are many well-formed formulae for which a set cannot exist. At the surface, this may look like that within logic we have the ability to produce things that do not exist, implying that logic may exhaust a greater domain than ontology. But the method of stratification clears this ambiguity. What matters to logic is the result of the last operation: operation of derivation. The seemingly larger domain is the outcome of the first and the second operations. Therefore, for example, an expression such as the self-belonging set, which is the basis for the Russell’s paradox, is filtered out (actually as a non-well-formed expression during the formation operation).

19 We are following Badiou’s definition of equal sign. A first order logic with equality is usually taken the equal sign as a primitive in the system and has a number of axioms associated with it, of which reflexivity is one of them (it has more axioms than the ones enumerated above). It is worth mentioning that in certain interpretations of first order logic equality may not be a primitive logical symbol. This logic is referred to as first-order logic without equality. If an equality relation is included in the signature, the axioms of equality must now be added to the theories under consideration.
its negation \( \neg l(x, x) \), which as we saw are produced by the function of concatenation in \( S \) and are slated as well-formed by the function of formation in \( E \). But the function of derivation dispenses them separately into \( T \) and \( NT \) respectively.  

Now let say that we come up with a new mechanism \( M_4 \) that adds to \( M_3 \) a predictive constant \( 0 \) we are going to define as follows. 

Let say in our system the function \( R(x, y) \) expresses that variables \( x \) and \( y \) are related within the system. Let us also assume that \( R \) is reflexive, that is \( R(x, x) \) states that whether variable \( x \) has a certain relationship \( R \) with itself: ‘to be linked to itself by the relation \( R \)’. Let us name the latter function (whether a variable is linked to itself through relation \( R \)) as \( Ar.R \), and accordingly \( Ar.R(x) \) means that \( x \) has the relation \( R \) to itself. 

Now, let say that instead of the first order logic we operate in second order logic, in which we can now select over not just variables but also functions. In that sense we can define the above definition in the following way:

\[
\forall R, x: Ar.R \in R(x, x)
\]

Given the definition above it is easy to see that how \( l \) is such an \( R \) according to the axioms. That is, the axiom of equality (identity) will allow \( M_3 \) to derive \( Ar.l \) and will not derive \( Ar.\neg l \). Let’s define 0 predicate as the following:

\[
0 = Ar.\neg l
\]

Or in other words:

\[
\forall x: 0(x) \iff \neg l(x, x)
\]

Now, the predicate 0 is an inscription that is accepted by \( M_4 \), due its well-formed-ness, and therefore it is in \( E \), rejected by \( M_3 \) because it cannot be derived, and added back again to \( T \) by \( M_4 \). What \( M_4 \) did was to add a mark to \( T \) for a non-derivable relation.

The zero marks in \( M_4 \) (in predicative form) not the lack of a term satisfying a relation but rather a relation lacing in \( M_4 \), the relation \( \neg l(x, x) \). We must nevertheless add: if the relation can be lacking in \( M_4 \) it is only insofar as it figures in \( M_4 \). 

How is 0 predicate derived here by Badiou different from 0 term derived by Frege (and used by Miller)? The difference is that the latter marks the lack of a term and the former marks the lack of a predicate. Frege’s version of zero states that there is no term \( x \) that can satisfy the negation of Leibniz identity principle, which as we saw is something that is fraught with metaphysics. 0 as predicate however mentions that a predicate that exists in one stratum \( (M_4) \) is erased from another stratum \( (M_3) \), for which we are going to devise a mechanism \( (M_4) \) to add a mark as a trace of this erasure. This is not just a zero sum game. Lack of a term and lack of a predicate are totally different things for one important reason: term (especially in Frege’s use) is a non-logical artefact whereas the predicate \( Ar.\neg l \) is an artefact of logic produced by \( M_4 \) and ratified by \( M_3 \) – we’re still well within logic: 0 is not a mark of what logic lacks, it is but what logic produces to trace the lacking of a mark within its extendible stratification.

[Science is] stratified in such a way that no lack is marked in it that does not refer to another mark in a subjacent order differentiated from the first.

Here we should emphasize two points, very important to the overall argument that Badiou is producing. In number of places in this short essay Badiou emphasizes that we should differentiate logic from the discourse about logic. The discourse about logic, which is usually used for pedagogical reasons, provides intuitive or commonsensical conceptions that are foreign to logic itself.

Like Lacan’s accounts of Gödel’s theorem and the semantics of implication, Jacques-Alain Miller’s discussions of Frege and Boole are ambiguous in that they combine, simultaneously and indistinctly, what pertains to the effective construction of a logical mechanism with what pertains to the (ideological) discourse through which logicians represent their constructions to themselves.
This criticism, which goes to the heart of Bachelardian theory of epistemology as a non-empirical endeavour, tells us that we should bracket out common sense and empirical concepts from the scientific domain, logic included, and differentiate the discourse about a discipline from the discipline itself. There is a convenient way that logicians speak about their theories, but when it comes to logic as a discipline, they abandon the niceties of the discourse and stick to what is the discipline itself works with. Practitioners and people whose professions do not involve the direct treatment with a particular discipline (logic or science) are the usual victims of the discourse, and err discourse for discipline. Philosophers are the prime example of such victims, and ideological recapture/representation is what this error produces.

Gödel’s theorem is a very famous case for such confusion between discourse and discipline. As we formerly saw there is nothing in the incompleteness theorem that speaks about lack in the predicate logic, or first order logic or arithmetic. It instead shows that language in a predictable way falls short of calling out, or deriving, all true statements. The logical result of this incompleteness theorem is in fact Cohen’s generic procedure, which embraces Gödel’s incompleteness results to show how we can constantly extend a consistent/semi-veridical ground model (an initial denumerable set) by forcing an indiscernible (or generic set). In contrast to Cohen’s use of Gödel’s theorem for example, which we may categorize as a legitimate, that is disciplinary, use or extension of this theorem by a logician who remains within the discipline, Lacan usage of Gödel in Badiou’s eyes, is illegitimate and influenced by misconception of what the incompleteness theorem really means. Lacan’s misconception states that this theorem proves the lack in the Other, which is the language or the overall battery of signifiers, and thus there is a need for an (reflective) element that has to constantly suture the Other, thence the role of the duped (and foreclosed) subject(s). The proper recapture of Gödel is by Cohen, whose main underlying and enabling thought was the prime example of such victims, and ideological recapture/representation is what this error produces.

To deploy the concept of suture in the very place where it is inadequate (mathematics), and to conclude that this concept enjoys a universal legitimacy over discourses by exploiting scientists’ conflation of their own activity (science) with its (ideological) representation, is to reflect science in ideology: it is to de-stratify it so as to prescribe to it its lack.25

Another noteworthy point in what Badiou presents pertains to the stratification of logic. We saw how with this stratification, logic from within itself can create abstractions that produce conceptual tools to address what it needs. But the question is whether there is a way to produce a logic of stratification itself? In the footnotes and in passing Badiou names two logicians who have attempted to answer this question: Wilfred Quine and Hao Wang.26 Quine attempted an axiom of reducibility to flatten out the strata to a single stratum and conversely Wang created an ‘expansive’ system $\Sigma$ to traverse the strata. But, according to Badiou, both attempts have failed. This failure means that a single meta-logic for the logical stratification does not exist. He captures the meaning of this failure as follows:

For our part, we are convinced that the stratified multiplicity of the scientific signifier, which is inherent to the process of scientific production, is irreducible to any of its orders. The space of marks does not allow itself to be projected onto a plane. And this is a resistance (or limitation) only from the viewpoint of a metaphysical want. Science wants the transformation-traversal of a stratified space, not its reduction.27

The effort to create a single theory that rules over the stratification of science is itself emitted from a metaphysical want – the desire to totalize. The discipline of science is of the order of infinite, and deals with the order of infinite, whose totalization is an impossibility. This yet points to how even in his early career Badiou’s theoretical conception is likely imbued by the thought of infinity.

We began this inquiry with reviewing the dashing intellectual thrust made by the editors of Cahiers announcing a new unified theory of discourse. At the heart of this new unified theory was the announcement that we should look at the action of the structure in the presence of a reflective

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25 Badiou 2012-A, p.173
26 Hao Wang (1921-1995) was one of the few confidants of Kurt Gödel. He was also Stephen Cook’s PhD thesis supervisor.
27 Badiou 2012-A, p.171
element. As seen earlier in this chapter the abrupt introduction of a reflective element into the language of structuralism could be only justified when this element is seen as a primitive, as nowhere in this theory there is any assumption of the presence, let alone the action or impact, of such an element. Seen as a primitive, such as it is, Miller derived a number of intriguing properties not present otherwise in traditional theory of structuralism: the function of misconognition, the imaginary and real registers, the lack and the suture, metonymical causation to name a few – in the center of all which there is a reflective element implicated by the structure and framed in such a way that its principle element is of non-identity. However, while Action of Structure talks about the theory, it is not the theory itself; Action of Structure can be only understood in my view by reading Suture, as it is only in the latter work that we find a derivation of the concept of subject in this theory. We see in Suture Miller’s appropriation of Frege’s arguments to establish the role of the subject in the number theory, but we can understand the true scope of this derivation only by looking back at Action of Structure. So, while Suture derives the subject, Action of Structure uses it to for building the overall scope of the program.

It is only when we look at both works by Miller that we can properly understand Badiou’s scope of rebuttal: from one hand, he has the task of countering the misconception inherent to both Frege’s and Miller’s derivations. Based on what we have seen Badiou is establishing the fact that there is nothing within the foundation of mathematics that requires an element of non-identity. Secondly, neither mathematics nor logic (nor science in the way demarcated by Bachelard) is sutured to anything outside of it: they don’t need something to bootstrap them and get them going – like the way Frege’s conception of number required bootstrapping by a recourse to the empirical version of the Leibniz law of identity. Science is foreclosed to anything outside it, it has a lack of lack, and this lack is not a lack itself – there is no trace of lack. Thirdly, science is infinitely stratified, which allows production of abstractions inside its realm, without needing to recourse to any theory or discourse outside of it – stratification whose strata are subject to a law that derives formulaically their depth and breath.

The immediate conclusion of this is that there is no subject of science: “science is a pure space, without inverse or mark or a place of that which it excludes.”

As also Tom Eyers has remarked, it is ironic that Badiou should choose ‘foreclosure’ a psychoanalytical term to describe science as something about which “psychoanalysis has nothing to say”. But what is looming under this term goes well beyond the psychoanalytical concept of foreclosure. The founding role that this term is supposed to elicit in Badiou’s work clearly illuminates the traces of the axiomatic orientation of thought, and while the term foreclosure implies closed-ness and protectiveness, logic, science and mathematics enjoy much openness and bountifulness. That is precisely the sense of positivity that the axiomatic thinking provokes. It grounds the thought based on a finite set of circumscribed decisions or ideas, not to the circumcision or foreclosure of thought and its possibilities, but to free the thought to explore possibilities in ways not otherwise possible. Axiomatic thought is what allows science to grow on its own merits alone: foreclosure in this case is not prohibitive in any sense. On the contrary, it is the founding principle of something productive and affirmative: the mechanism of production, partly in exhibit in Mark and Lack, upon which science can produce its signifiers, expressions, and abstractions according to its internal laws and its founding decisions. In that sense, although Badiou does not make any note of the axiomatic thought, we think a retroactive reading of this work, under the light of his mature oeuvre, leaves no room for doubt that Badiou is embarking on a project to juxtapose the axiomatic orientation of thought against the theory of discourse whose roots are in structuralist humanities: linguistics, anthropology, psychoanalysis, and last but not least historical materialism.

As the unified theory of discourse claimed to recover the repressed Truth in science and give a unifying voice to the discourses of overdetermination, in Badiou’s eyes at the time, it is evidently tormented by the same traumatic core as the philosophy itself – it attempts to bestow to science what it does not need and what it does not want: the repressed Truth of science, the Subject.

We can claim in all rigour that science is the Subject of philosophy, and this precisely because there is no Subject of science.

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28 Badiou 2012-A, p.171
29 Badiou 2012-A, p.172
30 Eyers 2013, p.87
31 Badiou 2012-A, p.172
32 This ‘liberating’ aspect of axiomatic thinking is mostly discussed by Albert Lautman in Mathematics, Ideals, and The Physical Real, Continuum International Publishing Group 2011.
33 Badiou 2012-A, p.173
Ontology of Discipline and Epistemology of Discourse

In the last section of Mark and Lack, partially titled as Alternating Chain of Science-Ideology, Badiou uses the consequences of Gödel's theorem to clarify a significant differentiation that exists between two 'positions' that ideology could occupy in relation to science. The position that has been discussed up to now pertains to discourses that are outside of science proper. Badiou captures the intent of this category of discourse as such:

The (metaphysical) project which, following Hilbert, enjoins every formal system to seal itself around the internal statement of its own consistency.34

These discourses go by different names (e.g. metaphysics) but in the context of the discussion Badiou is undertaking here we choose to call them epistemological discourses. Epistemology therefore refers to those discourses that have science as their subject but they do not produce, nor do they claim to produce, scientific theories themselves. In all their forms, epistemological discourses attempt to find from outside of a given discipline the unifying principle according to which the discipline can be defined and organized. Epistemological discourses, in that sense are transcendental to disciplines they study. Badiou uses Husserl's treatment of mathematics as nomology as an example of such a discourse. Being nomological here implies that the domain of mathematical objects can be exhaustively defined – as a formal system, mathematics is closed, saturated – in addition it means that the progression of the system by establishing different levels can take place without contradiction.35 Nomological definition implies that the technique establishing something like a meta-mathematics can be stopped at any time, once the increased facility permits statements about reality to be obtained. These are assertions none of which belongs to mathematics proper. They belong to a discourse that occupy a position outside of mathematics. They are discursive assertions about a disciplinary practice.

In contrast to this there are certain assertions about a given discipline that are part of the discipline itself. The assertions are not part of the disciplinary theoretical body but nevertheless they are considered part of the discipline. Two recent famous examples of these in mathematics are Hilbert's programme and Principia Mathematica both of which are programmatic projects that make assertions about structure and nature of mathematical theory in general. Badiou captures the nature of these programs as such:

The project which, by means of the completely controlled re-arrangement of a logistical system, claims to exhaust what otherwise presents itself according to the opacity that results from a history: let us call this 'intuitive' arithmetic.36

We can also say that programmes like Hilbert's or Principia Mathematica are also representations of mathematical discipline, but representations that are intra-disciplinary, representations that are immanent to a given discipline – which Badiou calls the 'intuitive' part of a given discipline.

Therefore, according to Badiou, there are two representational domains: one is outside and one is inside disciplines, and they correspond to two aforementioned positions of ideology.

Now one way that Badiou's critique of the project of unified theory of discourse can be understood is that he does agree with the overall intent of the programme to identify and reveal ideology but both the position he identifies for ideology and the method he thinks science itself is taking to confront the ideological representation are different than Miller's proposal. The unified theory of discourse targets the transcendental position of ideology and its method is to devise a unified theory of discourse that aspires to science in order to remain non-ideological.

What we are proposing is that Badiou is critiquing both the target and the method of this theory. He is critiquing the target because the ideological representation of science outside of the science is not what affects the practice of science. What matters to the practice of science are ideological representations that are immanent to it: according to the above differentiation the intra-disciplinary representations. Badiou clarifies this in the following passage:

But that a crisis in the (ideological) representation of science can induce a (positive) reconfiguration of science itself should not surprise us, given that the material of science is, in the last instance, ideology, and that an 'a priori' science by definition deals only with those aspects of ideology which represent it in the latter: a science continually breaking with its own designation in representational space.37
What has a positive influence on a science, its advancement for instance, is when science breaks with its own representative designations, that is, with what a science thinks about itself (as opposed to what philosophy thinks about it). In addition to this Badiou also critiques the method of the unified theory of discourse, not just because Badiou has been able to unwind the metaphysical core of its notion of subject that is the linchpin of this theory, but because at the end this theory is just another discursive tool and discourse is not how science ploughs through its ideological obstacles. So, what is it that science does that has been historically so effective in constantly removing from its path its own ideological designations? The answer is the disciplinary engagement: it is the discipline itself whose practice constantly breaks away with how the discipline is represented inside the discipline itself, whose term for Badiou is ‘Formalism’ in contrast to the representation of the discipline, which as we saw before Badiou calls ‘Intuition’. So, the faceoff that matters to science is the faceoff between formalism, “an entirely coded scriptural artifice”, and intuition, “the immanence of a historico-institutional discourse living off the abbreviations, equivocations, and univocal smoothing of an inoffensive mass of ‘normal’ signifiers legitimated by custom and practice” (ibid), and thereby a wholly intra-disciplinary faceoff.

And yet again, Gödel’s theorem plays a key role in demonstrating this faceoff between the intuition in science, which involves “certain ambiguities produced in language by the (ideological) concept of Truth”, and “formalism’s fidelity to the stratifications and connectivities at work in the history of the science, insofar as they expel from the latter every employment of the True as (unlimited) principle.” In contrast to this Gödel’s proof also had consequences for nomologist conception of mathematics as well. It proved that the mathematical content is not nomological, that is mathematics is not tautology. Mathematics can provide material that is not the result of analytical manipulation of axioms. As Cavailles mentions, Gödel’s statement albeit undecidable, still represents an increase of knowledge. That the undecided statement is legitimate implies, for Cavailles, that formal systems possess their own proper content, different from experiential content. In addition, that the expansion of a formal system does not take place in a predictable way; rather, based in its own sort of content, formal systems exhibit their own sort of necessity. Together, these consequences proved that formal systems such as mathematics are not closed systems (as Husserl had postulated). Nomology can thus be perceived as the epistemological principle around which Husserl could conceptualize mathematics as such. By prescribing or uncovering an inherent limitation in the ability to completely identify the true statements in a formal system Gödel proved that mathematics is not a nomologist system. But for mathematicians, unlike Hilbert’s programme, whether or not mathematics is a nomological system or not was never a formalist concern.

Badiou thereby interprets formalism as engagement, an operator of fidelity operative inside science. The general theory of discourse is to show how the discourses of overdetermination break away from ideology. Badiou’s rejoinder here is to show that the proclamation of the latter still leaves us in the realm of ideology, because it is still speaking of the discourse of science as opposed to the science itself, or in the terminology that we used here, it still posits the discourse as something separate from the discipline. The discipline of science breaks with itself: when science encounters its limitations, it treats these limitations as ideology and breaks away with them. With Miller, we always speak of ideology until science allows us to spot the ideological misconception. For Badiou on the other hand we have scientific thinking and only when we try to think that why or how a science is a science instead of to continue making science that we fall into ideology. So, the development of sciences involves the critique of the philosophical idea of science. For Badiou, given the way he criticizes philosophy and the way in which he brings in the alternating chain of science and ideology, the point he is making is that we don’t need to look between science and something else in order to see the oscillation between science and ideology. Within science we have scientific thinking, while at the same time we have scientific ideology: the case in point is what is happening between Gödel and Principia Mathematica, the latter standing for an ideological recapture of mathematics, inside mathematics itself. What we said earlier in this chapter regarding Church/Turing and Cook/Levine theorems are also examples of such a break from Hilbert’s programme, which equally stands for an ideological recapture of mathematics, and again within mathematics proper. That is why Badiou says that through science we learn that there is something un-sutured. Opposite to the claim that science is the science of suture, we learn that through the scientific practice we get something that is not a hiatus between ideology and science. Scientific practice is a constant separation between formal means of thinking and formal means of representing. In this sense science continually breaks with its own designation in the representational space. Therefore, rather than staging the debate between science and ideology at the level of discourse, Badiou brings back this de-
bate to the level of discipline: the level of production and practice of true science. In *Mark and Lack* Badiou has shown us that logic is a practical endeavour: the practice of recognizing, concatenating and deriving marks, traces and expressions. With this he brings the debate over logic from the level of discourse to the level of discipline. It is within the discipline itself that a division with ideology takes place. This division then concerns the being of science, and it is not a division between science and something else.

Now why is this important? This distinction anticipates three significant themes in Badiou’s later work. The move from discourse to discipline means the move to the very entailment of things, to the level of their being, and thereby designates a move from epistemology to ontology. From this point of view the second theme is motivated by the recognition of scientific process as a process of fidelity. And finally, the fact that the hiatus does not take place between two realms but between a realm and itself, one could see how it gives rise to the theory of event. In addition, in the way Badiou presents Gödel’s theorem as a nexus of interconnected commitments to the logical construction as well as the confrontation with tacit or declared ideological positions suggests in an implicit form a sort of engaged theory of subjectivity, and although Badiou in *Mark and Lack* is still within the Althusserian world of epistemology, his viewpoint here seems to anticipate the next chapter in Badiou’s development to fully engage with the theory of the subject. This is what we will take up in the next part and show how Badiou from here will undertake the project of the subject.

**Conclusion**

In an interview with Peter Hallward in 2007 Badiou mentions that *Cahiers* project was inspired based on a certain understanding of structuralism as “a certain Lacanian interpretation of scientism”.41 They sought to find in scientism itself, in extreme forms of formal thought, something to support the Lacanian theory of the subject. In my view that is why Miller’s text ‘Suture’ is programmatic. It is a fundamental text in this regard, because this is the text that manifests the synthetic genius for which Miller must undeniably be recognized: he shows that for Frege the logicist reconstruction of the theory of numbers conceals an operation which can only be interpreted as the operation of a subject. I would say that this was the general orientation.43

In early 60’s the general problem of the relation of structure and subject were raised anew among the generation of young philosophers in France. In the model upheld by Sartre’s existentialism it was subject and consciousness that was considered as the primary in this relationship and all forms of structure were engendered on the basis of an absolutely simple and initial determination, which was practice. In early 60’s Sartre’s generic philosophy lost its grip:

We were no longer in a position to believe in it. That is to say, we were no longer able to believe in the engendering of the general system of formal structure on the basis of the simple intentionalities of consciousness.44

The alternate approach was a commitment to structure first but to enable the structure to harbour the element of the subject. As Badiou mentions, Lacan was the one who proposed the alternative to Sartre. I think that was Lacan’s major philosophical influence. That is, the ability to bring together, in a thoroughly unusual way, a theory of formal structures, which he developed as the logical theory of signifiers, and a theory of the subjective adventure.45 This is a very important assertion, and for reasons that cannot be developed in this work, none of the French structuralists, and in particular Althusser, were able to make such a proposal, mainly due to their lack of commitment to the category of subject. The project that was taken up by *Cahiers* was then to elicit what was already present in Lacanian theory of the subject, something that *Cahiers* called the unified theory of discourse. This theory started by engaging in the most extreme formal rigour and by taking up the intellectual power of mathematics and logic but at the same time, as we saw earlier, showed how an element of indeterminism must be sutured and present for the proper functioning of the structure, at the center of which there are two operators of metaphor and metonymy. Badiou around this time was a member of *Cahiers* and more importantly, to his own words, completely committed to the agenda that brought this group together – to raise anew the question of the relation between structure and subject post Sartre, and although he undermines *Cahiers* manifesto, by no means he ever abandoned the original project.

The fact is Badiou’s work in this era showed one thing and that is *Cahiers* and psychoanalysis commitment to science is not thorough enough. The introduction of indeterminism and non-identity to the science.

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41 Badiou 2012-B
42 Badiou 2012-B, p.277
43 Badiou 2012-B, pp.277-278
44 Badiou 2012-B, p.278
45 Badiou 2012-B, p.278
ence perceived as the realm of self-identity is abrupt and ideological. We will see that it is exactly by deepening this commitment to formalism and determination that Badiou finally opened the path to indeterminism and non-self-identity, of the entire situation of being. Non-identity is the law of being not of the subject.

But this path goes through tumultuous times through which praxis yet again gains a radical priority for Badiou. At the end of this era, in early 70’s, the question of immediacy of praxis and the role of the subject finds a heightened urgency for Badiou, through which he comes to rethink the relation of structure and subject. And this will take us to the next part of this work.

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