The Logic of Lacan’s Not-All

Dominiek Hoens

Abstract: This article offers an analysis of Lacan’s notorious formulae of sexuation. Lacan develops these formulae on the basis of the classical Aristotelian square of opposition. Nonetheless, attempts to localize the formulae within such a square, lead to all sorts of logical inconsistencies. Therefore, the first conclusion of this text is that these formulae cannot be placed on the four, relationally-determined corners of a square. A second thesis starts with the simple observation that Lacan’s formulae consist of two pairs of two equivalent propositions, as a result of which there is, at first sight, no difference between the two, so-called masculine and feminine sides of Lacan’s diagram. The difference, however, does not concern a negation (e.g., \( p \) and \(-p\)), but a difference in use of the quantifiers. This goes for both the universal, \( \forall \), and existential quantifier, \( \exists \); in this article only the former is discussed. Derived from a critical examination of works by Peirce, Frege, and Blanché, Lacan introduces a distinction between the extensional and intensional interpretation of a universal quantifier. This distinction allows to interpret \( \forall \) in ‘\( \forall x \Phi x \)’ extensionally (\( \Phi x \) applies to all \( x \)) or intensionally (\( x \) is essentially \( \Phi \)). This distinction implies that there may be no exception to \( \forall x \Phi x \), but also that one cannot exclude that there is an \( x \) to which \( \Phi \) does not apply as an identifying feature. Moreover, it divides the all into an all and a not-all, where the latter does not negate the former, but reveals the illusion that ‘all’ subjects are defined necessarily by the function \( \Phi \).

Keywords: Formulae of sexuation, Aristotle, square of opposition, Peirce, object a, extension, intension

Introduction

Although Lacan’s formulae of sexuation have often been discussed before, commentaries on the logic pertaining to them are rather rare.2 The reason for this, unsurprisingly, has to do with Lacan’s undidactic presentation of them. Simply put, there is no single écrit or part of the

1 I learnt very early on that logic is capable of incurring the world’s odium.1

Seminar that summarizes in an accessible way what is at stake in them. For a discussion of the formulae, many readers have recourse to Seminar XX, but this seminar actually the last one in a series starting with the unpublished Seminar XV (1967–1968) – with, as we shall see, antecedents dating back to at least the early sixties –, during which the logical square is being constructed, detours included. Therefore, in order to articulate what is at stake in the formulae one needs to consider the seminars preceding Seminar XX. Which is not an easy task, for the formulae do not concern just one aspect of psychoanalytic theory – sexuality – but are related to most if not all of Lacan’s fundamental ideas regarding psychoanalysis: its relation to science, its ethics, discourse theory, the question of desire, enjoyment and love, the difference between speech and writing, etcetera. Therefore, this article’s aim cannot be but a modest one, that is to situate the formulae within the broader context of Lacan’s work and to detail their logic.

**No ‘relation’?**

The problem that the formulae tackle can be put in fairly simple terms: if there is sexual difference, and if this difference is of such a nature that it does not allow for a relation (rapport) between the two sexes, how to conceptualize this difference? In what way do the feminine and the masculine differ from each other, so to make absent any sexual relation between them? Given the everyday experience of love and desire between the two sexes, leading to all sorts of phenomena that can hardly be named other than relational, how to make sense of Lacan’s famous dictum ‘il n’y pas de rapport sexuel’?4 Does such a statement amount to more than an affirmation that there is no sexual relation, moreover, that precedes it.

Besides this, the so-called ‘mathemes’ Lacan provides his readership with, are not unlike mathematical formulae in using letters (instead of words open to diverging interpretations), but also share with them the quality of establishing relations between those letters.7

And, finally, the grid onto which the formulae of sexuation are inscribed, derives from Aristotle’s logic, which, again, consists of relations between different formulae. This becomes clear if we take a look at the Aristotelian square of opposition, containing four formulas. In themselves they express already a relation between a subject (S) and a predicate (P) – differing qua quantity (all or some) and/or quality (affirmative or negative), allowing for four possible combinations, A (all +), E (all –), I (some +) and O (some –) – but they become logically even more relevant through the relations they entertain with one another. These relations facilitate simple derivations – from “All men are mortal” (A), one can conclude that “Some men are mortal” (I); if “Some men are mortal” is true, then “No men are mortal” (E) is definitely false, etcetera – but also more complex forms of syllogistic arguments.4

![FIGURE 1: Aristotle’s square of opposition](image)

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4 The first literal occurrence of this statement can be found in Lacan 2006b [1968–1969], 226, although there are precursors such as “Le sexe, dans son essence de différence radicale (…)?”, cf. Lacan 1964–1965, 19 May 1965.
6 Cf. Ferdinand de Saussure 1966, 120: “[t]In language there are only differences without positive terms.”
7 See Burgoyne 2003 and Corfield 2002 for a critical examination of these mathemes.
8 Kneale and Kneale 1962, 67ff.
9 Parsons 2017. It should be noted that one will not find this ‘square of opposition’ in Aristotle’s works, the diagram was constructed by the 2nd century philosopher Apuleius, based on his reading of Aristotle’s Organon. Although the square fell into disuse in the twentieth century – due to the introduction of the formal logic developed by Frege, Russell, and many others – it continued to be discussed in the writings of Jacoby 1950, Blanché 1957, 1966 and others, eventually leading to a complete research programme on logical geometry; see [http://logicalgeometry.org](http://logicalgeometry.org). Jean-Yves Béziau wrote an accessible and entertaining article on issues related to the square and its more recent transformation into a hexagon. Here, one paragraph is worth quoting at length: “The problem of the O-corner is quite different from the problem of the I-corner. The question is not a wrong-name problem but a non-name problem. This has been pointed out especially for the case of the quantificational square: it seems that there are no natural languages in which there is a primitive name for ‘not all’. In such situation, linguists speak about ‘non-lexicalization’. A radical view would be to argue that if there...
The relations between the formulae can be defined as follows: “Two propositions are contradictory iff [= if and only if] they cannot both be true and they cannot both be false. Two propositions are contraries iff they cannot both be true but can both be false. Two propositions are subcontraries iff they cannot both be false but can both be true. A proposition is a subaltern of another iff it must be true if its superaltern is true, and the superaltern must be false if the subaltern is false.”10

So, at first sight, the logical square of opposition seems to be a rather unfortunate choice for any possible explanation of the thesis according to which there is no sexual relation. This becomes even more problematic if one takes the appearance of Lacan’s version of the square into account.

**FIGURE 2: Lacan’s formulae of sexuation**

One discerns two sides, a left and a right one, textually clearly identified, respectively, as the male and female side.11 So, in addition to the relationality inherent in the square of opposition, Lacan’s re-elaboration of it seems to involve the most classical of binarisms: the one and its other, separated into two distinct, juxtaposed halves.

This not only runs counter to what seems to be at stake in the formulae – a non-relational difference – but should also be situated against the background of a more general problematic. First of all, a constant theme within Lacan’s work concerns the object-pole of the subject-object relation, and more in particular that this object should not be understood as the thing that would make one complete. Surely, the object a is often considered as a lost object, entailing the idea that it could be found again or somehow recuperated. Yet, the proper Lacanian additional qualification of this lost object is that it is structurally lost and that the classical Freudian idea of a paternal castration threat forbidding access to the incestuous maternal object, is nothing but a prohibition of what is anyway impossible qua object of desire. In that respect, the Lacanian approach of desire is at odds with a long, dominant Western ontological, ethical and political tradition – from Aristophanes’ myth about human beings as divided into two halves desperately searching one another in order to reconstitute a whole, up to a certain das Wahre ist das Ganze – which reserves an important place for what one may call ‘plenitude’ or ‘wholeness’. The issue, however, is to conceptualize this structural incompleteness both against and within a tradition that advocates one or the other version of ‘wholeness’. One can state and repeat that the object is structurally lost, proclaim the subject to be a divided subject etcetera, but the challenge remains to argue for this in a theoretically solid way.

With the formulae, the issue can be put as follows: how to conceptualize a difference that makes any universalization impossible, a difference as radical that it is not based upon a common ground or is played out against the background of a unifying domain? This problem seems to have preoccupied Lacan from the very beginning up and until the last phase of his work. In the imaginary, the basic operation is (mirroring) identification, through which all are equal. Sexual difference on this level, formalized as -φ, is about having or not having a penis and suggests a potential complementarity between those who have and the have-nots.13

Within the symbolic, difference is the operational principle, yet this difference is ultimately guaranteed by one specific signifier, the phallus. As the signer of lack – the negation at work within the symbolic system – the phallus both establishes the link with the signified or (sexual) meaning and bars any definite access to it. Put differently, in Lacan’s structuralist account of the unconscious the Saussurean sign becomes stripped of its unity, relocating the signer and the signified to two separate realms, kept apart by a bar. This function of the bar, i.e. to prevent the two realms from blending into one, is taken up by the phallus. Although this allows for a take on the subject as lack or as marked by the bar of castration – hence the notation $ for the barred subject –, within this frame of thought, sexual difference seems to be a mere symbolic issue of a division into two positions. Regarding the phallus different (symbolic) positions are possible14, but that does not change the basic conception of the subject

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10 Parsons 2017, my italics.

11 As noted above, it took Lacan several years to construct this square and hence there are different, incompatible versions of it. Here, the one discussed is the final version, included in Lacan 2018 (1971-1972), 95, 118, 178, 183 and Lacan 1998 (1972-1973), 78.


13 Women, according to Lacan, are not sexually desirable for a man because of their genitalia, but because of their motherly physical characteristics, breasts and hips; see Lacan 1998 (1972-1973), 7 and 35.

14 Cf. Lacan’s earlier work, according to which masculinity and femininity are different with respect to each other (i.e. in a relational way), in the sense that the one and the other involve, respectively, having the phallus (on the basis of initially not having it and of possibly losing it) and being it.
qua subject of the phallic law. As a subject of the symbolic, the subject may be thoroughly sexual, but this sexuality is inscribed into the symbolic and is nothing but a series of possibilities to situate oneself with regard to the phallic, based on a primordial affirmation of a symbolic order organized by the phallic law.

In Lacan’s later work phallic determination remains the basic idea when it comes to conceptualizing desire and enjoyment. Figure 2, the table of the formulae of sexuation, includes one (and only one) function, the phallic function. Therefore, if one is looking for ‘a jouissance beyond the phallic’14, a supposedly exclusively feminine enjoyment, one wonders where to look for this in a (logical) universe made up of one function (Φ) and one variable (x).

### The real difference

Influenced by Alexandre Koyré on this point, Lacan considers mathematics to be the defining characteristic of modern science’s modernity and does not refrain from introducing it into his theoretical apparatus. However, it is only logic that provides a basis for mathematics, examining its method and its proofs, including inferences such as ‘if... then...’ and connectives such as ‘and’ and ‘or’. It is a this point that one can situate a domain, shared by both psychoanalysis and modern science, namely logic. As Lacan put it early on: “[...] all what psychoanalysis is about, is of the order of language, that is, in the end, a logic.”15 At a later stage of his teaching, and most clearly within the context of the formulae, logic is considered as providing an access to the real, for “[...] it is only because of logic that there is an access to the real.”16 This may sound surprising, for wouldn’t one expect the ‘real’ – certainly if one distinguishes it from ‘reality’ – to be something outside and different than the manipulation and connection of symbolic notations, such as p Φ q? This becomes more intelligible if one adds that Lacan equates the real with the impossible. A passage in Seminar XVII is unambiguously clear about this: “[...] the real is the impossible. Not in the name of a simple obstacle we hit our heads up against, but in the name of the logical obstacle of what, in the symbolic, declares itself to be impossible.”17 In brief, the real is the impossible and one encounters it as a ‘logical obstacle’ in the symbolic at its purest, that is, logic. As he puts it in Seminar XIX: “[...] the real I’ve been speaking about is accessed via the symbolic. We access this real in and through the impossible that is defined only by the symbolic.”18 This may provide some answer, but also shifts our question to: what is this ‘logical obstacle’, this impossible real within the symbolic, qualified as an impasse of formalization?19

‘Obstacle’ is not a novelty in Lacan’s work. As Jacques Derrida’s pupils point out in their The Title of the Letter20 – a work praised and repeatedly referred to in Encore – the notion of ‘instance’ in Lacan’s early text, ‘The Instance of the Letter’21, should be related to the Aristotelian notion of enstasis, meaning ‘objection’ or ‘counter-argument’.22 In ‘The Instance of the Letter (and elsewhere), this objection is clearly a phallic objection or obstacle, in the sense that the field of desire is characterized by a structural and constitutive lack, symbolically incarnated by the phallic signifier. The subject is not ‘one’ and fundamentally lacking, despite all the (psychoanalytic) myths about love as fusion or about, conversely, the birth trauma as the painful undoing of a unity with the maternal body. The phallus is, as noted above, the obstacle to any oneness, to any obliteration of difference (i.e. the endless metonymy of the signifier), or to any erasure of the bar separating signifier from signified. As Lacan reminds his public in Encore: “[...] the function of the bar is not unrelated to the phallus.”23 Despite its de-unifying function, the phallus is a bad candidate to incarnate the ‘logical obstacle’ Lacan is looking for. On the one hand,
because it is considered to be a function – which is not an obstacle, but on the contrary, a fundamental operator within a logical universe – and on the other hand, because the phallus may install lack and undo unity, but it does not seem to entail an exception to its universality. So, if there is an ‘other jouissance’, a feminine exception to the universality of the phallic function, and given Lacan’s avoidance of simply stating that women are not subjected to the phallic function – which would run counter to his fundamental definition of any subject as a subject of the signifier – what sort of exception are we dealing with? The latter is definitely not a simple negation, but rather an obstacle. And can we find this feminine jouissance beyond the phallus qua obstacle within logic, as Lacan seems to be convinced of? Lacan’s contention is that this obstacle is a symbolic obstacle, not in the sense of the phallus as a detotalizing function, but as a logical impasse. Lacan identifies two of these impasses, namely the ‘not-all’ and the ‘at-least-one’ (hommoinsun). Before going into a discussion of these impasses, we first need to acquaint ourselves with the domain within which these impasses occur, i.e. logic.

**Peirce’s quadrant and Frege’s judgement**

As mentioned above, Lacan’s formulae of sexuation are constructed in a dialogue with the Aristotelian logic and square of opposition. This logic was dominant from ancient Greek times until the end of the nineteenth century. Despite this dominance, most philosophers struggled with several issues pertaining to the square. The most notorious one concerns the so-called existential import of logical statements. If one states that ‘all men are mortal’ does this imply that ‘men’ exist? If so, how about ‘all unicorns have one horn’? The former statement is obviously true – it is, what Kant would call, an analytical statement, in the sense that the notion of ‘unicorn’ necessarily implies ‘one horn’ – but one may be reluctant to conclude that unicorns exist. The example of the unicorns may lead one to conclude that statements of the form ‘All S are P’ have no existential commitment. Yet, that does not solve the problem within the Aristotelian square, for as we have seen above, there is a relation of subalternation between the A- and I-corners. This means that if “All S are P” is true, then “Some S is P” is also true. The latter formula – the I-corner – explicitly suggests that there is an S (e.g. ‘unicorn’) if one holds ‘Some S is P’ to be true. In brief, it seems that Aristotle’s logic forces one to hold certain statements to be true, although one might not want to do so.

The problem of existential import was eventually solved elegantly by Charles Sanders Peirce. In his chapter of *Elements of Logic* dealing with the square, Peirce proposes the following division of the four basic propositions, A, E, I and O.

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27 \[\ldots\] it is not at all a matter of making one the negation of the other, but on the contrary of one standing as the obstacle to the other.” Lacan 2018 [1971-1972], 86; my italics.

28 \[\ldots\] the jouissance that people call by whatever name they can find, the other one, precisely, the one that I am trying to get you to approach by a logical pathway, because, as things currently stand, there is no other [than a logical pathway].” Lacan 1996 [1972-1973], 75.

29 Cf. Lacan 2001: 479: “Recourir au pas-tout, à l’hommoinsun, soit aux impasses de la logique.” Throughout this article pas-tout is rendered as pastout and not as pas-tout or as pas tous. This is in accordance with the former quote and stresses the unity of the negation and the quantifier ‘tout’. Surely, Lacan uses variations such as pas toux, but his first aim is to introduce a new quantifier, pastout, and not so much to state that ‘not all [two words] x...’ or that x is ‘not-whole’.

30 Which is certainly not Aristotle’s position on this issue, as his logic is clearly an onto-logic, i.e. statements of the form ‘all S imply the existence of at least one S.

31 Peirce 1933 [1883].

32 Ibid., 459.

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**FIGURE 3: Peirce’s diagram**

In the example Peirce uses, the subject is ‘line’ and the predicate is ‘vertical’. Remarkable here, is that the empty quadrant 4 of the diagram, is shared by both the A and E propositions. This means that universal statements, either positive (A) or negative (E), do not necessarily entail the existence of the subject term (in this case, ‘line’). They are both trivially true if their subject does not exist. Only propositions of the I and O form imply the existence of their subject term.

Peirce’s diagram may solve the issue of existential commitment, yet the price one pays for it is that the Aristotelian relations between the propositions disappear. Whereas Aristotle considered A and E to be contrary (cannot both be true, but can both be false), Peirce names them “incongruous or disparate,” for they are different, yet they can both be true (cf. quadrant 4) and both be false (cf. quadrant 2). Subalternation is no longer a valid inference (from A to I, or from E to O); I and O can no longer be called ‘subcontraries’. The only surviving relation is the one of contradiction (both cannot be true, yet one must be) between, respectively, A and E, and O and I.
For our purposes it is worth noting that Lacan uses this diagram throughout his Seminar, starting from the ninth, that is L'identification (1961-1962). In those sessions the repeated take on Peirce’s diagram consists in pointing out how the affirmative universal (A) is valid both when all S are P (quadrant 1) and when there is no S at all (quadrant 4). This leads to a reformulation of A into ‘there is no line which is not vertical’. This double negation makes explicit the operation required to actually arrive at a subject: either there is none, or one needs two determinations (or negations), namely two predicates, ‘line’ and ‘vertical’. The subject, ‘line’, only becomes identifiable if one adds a second predicate, ‘vertical’. Were one to isolate the pure subject (line), stripped of its verticality, then the subject is lost, and one shifts to quadrant 4: if there is no verticality, there is also no line. Interestingly, here the ‘impossible’ occurs for the first time: if the subject belonging to quadrant 1 were to ask ‘is there a line without being vertical?’, the answer it gets from quadrant 4 is ‘not possible’. In brief, the imagined exception (the ‘not possible’) of quadrant 4 not only confirms the rule of quadrant 1, but creates the rule of quadrant 1: the general identification of the subject ‘line’ with the predicate ‘vertical’ is only possible by excluding the possibility of the absence of the predicate ‘vertical’ (quadrant 4).

This ‘no line which is not vertical’ is true when there are either only vertical lines, or no lines at all. This basic structure of the A proposition — which defines the subject, for it says what it is and how one can recognize it — is approached in a different way by Gottlob Frege. Although Lacan’s emphases are similar to the ones regarding Peirce’s diagram, from the brief overview below we will gain one additional and crucial element that makes the formulae of sexuality readable.

### Referring to Frege

The introduction of Frege’s logic in the fifteenth Seminar, L’acte psychanalytique, occurs rather abruptly but is also no surprise. In the preceding lessons, logic already appeared, in particular the distinction between the classical, Aristotelian logic and the modern one, initiated by Frege, amongst others. The ambition of Frege’s Begriffschrift consists in purifying logic of any trace of natural language or what he calls the Sprache des Lebens. This attempt at stripping language of any ambiguous meaning effect or ‘content’, in order to solely preoccupy oneself with its form, is not a novel one. Already in Aristotle’s work — the Analytica Priora — the initial impetus to formalization is present, yet the application of (meaningless) letters remains rooted in the Greek language, with all sorts of odd and obscuring effects. Frege’s intention, however, does not consist in formalizing language to such an extreme that one would be left with a totally ‘empty’ formal language. He rather wants to develop a new language, apt for pure thought. In this new language concepts (Begriffe) are expressed, but only as a writing of logically well-formed sentences and of their interrelations. In this respect, his logic resembles mathematics, but as a method it is situated at the more fundamental level of elementary notions such as ‘and’, ‘if’, ‘not’ and so on. In general, the aim is to arrive at a Lückenlosigkeit der Schlusskette, that is to create a language that allows for a watertight connection between logical sentences. In order to do so, only two connectives are required, negation (Verneinung) and implication (Bedingtheit), the conditionalization of ‘if ... then ...’ clauses, added to the general form of assertions:

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\text{FIGURE 4: the basic form of an assertion}^{40}
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34 Lacan 1961-1961, 7 March 1962. See also: “Little Hans, who is as much of a logician as Aristotle, postulates the equation All animate beings have a phallus. I assume I’m addressing people who followed my commentary on the analysis of Little Hans, and who also remember what I took care to accentuate last year [that is in the Seminar on Identification, 1961-1962] concerning the proposition known as the universal affirmative, namely, that the universal affirmative is only meaningful in defining the real on the basis of the impossible. It’s impossible for an animate being not to have a phallus.” Lacan 2014 (1962–1963), 78.

35 On a formal level this argument is analogous to Jacques-Alain Miller’s reading of Frege’s theory of number: one only arrives at one (1), at something countable, by excluding an impossible concept and including it as an empty set, that is zero (0). See Miller 1966 and also Lacan’s notion of alienation: subjectivity is based upon a forced choice for one (vertical) out of two options, vertical or non-vertical. If one were to choose non-verticality, there would also be no subject. In that sense, the neurotic subject is a reply from the real (Lacan 2001, 459), for its symbolic subjectivity is based upon an excluded impossibility (the real), yet this exclusion needs to be reckoned with as cause if one wants to understand neurotic subjectivity, i.e. a subjectivity not fully coinciding with its symbolic identifications (or logical predicates); see also Lacan 1961-1962, 21 March 1962.


38 Frege 2014, x.

39 Ibid., x. Lacan reminds his audience of the contemporaneity of Freud’s psychoanalysis and Frege’s foundation of modern logic; cf. Lacan 2018 (1971-1972), 41. It is tempting here to connect Frege’s desire for a language without gaps (lückenlos) to Freud’s development of psychoanalysis, based on a taking into account of the gaps (Lücken) in the continuity of conscious mental activity. See, e.g., ‘Manuskript K’ (1896), which deals with hysteria and its primary symptom, i.e. the Schrecklussäuberung bei Psychischer Lücke; Freud 1986, 177.

40 Ibid., 24.
An assertion consists of a horizontal line (the Inhaltsstrich, content stroke), preceded by a small vertical one (which indicates that the sentence is an Urtei1, a judgement, meaning that one holds the assertion to be true). At the end of the sentence Frege places the function which applies to the argument (or variable) for which a concavity (Höhlung) is reserved in the middle of the horizontal line. This concavity is the place for what one traditionally called the subject. However, Frege’s reformulation of the subject-predicate logic into a logic involving functions and arguments has the advantage that no existential commitment whatsoever is made. Sticking to our initial example of ‘all lines are vertical’, ‘vertical’ and ‘line’ become, respectively, function and argument. If there are no lines, the concavity remains empty, which does not change the proposition’s truth. Moreover, and more clearly than is the case with Peirce’s diagram, there is no essential relation between argument and function. The function ‘vertical’ can be applied to any x that is vertical. In that respect, Frege’s logic is a purely extensional function. The function ‘vertical’ can be applied to any x that is vertical.

In Frege’s logic – not without reassuring them that this is tied up closely with the psychoanalytic clinic – to illustrate his definition of the subject as that what a signifier (S1) represents for another signifier (S2). In Frege’s rearticulation of Aristotle’s logic the bond between argument (S2) and function (S1) is loosened to the point that together they may form a valid logical proposition, but there is no longer any essential bond expressed between them. ‘Line’ (S1) may be the argument of the function ‘vertical’ (S2), but nothing prevents any other S1 to take up the place of the argument (= function), since the x of a logical proposition already seems to imply a well-defined collection of such x’s. Within the context of the formula of sexuation, this fundamental question is repeated when Lacan is commenting on the formula ∃x: “What is this x? I have said that it is defined as though by a domain. Even so, does this mean we know what this is? Do we know what a man is when we say all men are mortal? We learn something about him from the fact of saying that he is mortal, and precisely from the knowledge that this is true for all men. However, before introducing this all men, we only know the most approximate features, which can be defined in the most variable fashion.”

This observation, of course, does not impair Frege’s logic, but touches upon a more general issue which, as we will see, proves to be crucial for Lacan’s construction of the formulae of sexuation.

**On why the square is not a square**

Before presenting our reading of the formulae, we need to discuss one more logical aspect. Apart from Frege and Peirce, Lacan also pays tribute to Jacques Brunschwig, who published a technical article on Aristotle’s logic in the last issue of Cahiers pour l’analyse (1969), a journal founded by Jacques-Alain Miller, amongst others. In this article, Brunschwig points out an ambiguity in Aristotle’s use of ‘some’. ‘Some’ may mean ‘some, perhaps all’ – this is its so-called minimal interpretation, allowing for subalternation between A and I – or rather ‘some, not all’. The latter, ‘maximal’ interpretation corresponds to the ordinary usage of the word ‘some’, but logically it has quite a dramatic effect on the square of opposition. If one opts for the maximal interpretation of ‘some’ – ‘some, but not all’ – then subalternation between the universal and the particular is no longer possible. Instead of being able to derive the particular from the universal, A and I entertain a relation of contrariness (both can be false, but not true together). The relation between the two particulars, I and O, changes as well, from subcontrariness into equivalence, for if, e.g., one states that ‘some birds are black’ (I) – and here, ‘some’ means ‘some, but not all’ – then one implies that ‘some birds are not black’ (O), and vice versa.

There seem to be good reasons to relate Lacan’s pastout – i.e. one of the ‘logical impasses’ referred to above – to this maximal interpretation.

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43 Ibid., 90; see also Lacan 2016 [1975-1976], 6, and 184-185 for Jacques-Alain Miller’s clarifying note, including more references.
44 For a summary of Brunschwig’s article, alongside translations and comments on many of the other articles included in Cahiers pour l’analyse, see the resourceful website http://cahiers.kingston.ac.uk put together by Peter Hallward, Knox Peden et al.
of ‘some’. First of all, it forms an obstacle or objection to the universal proposition including the same subject and predicate; and what we are looking for is, as noted above, an obstacle to universality. Secondly, in most contexts the first meaning of this some is ‘not-all’; if one states that ‘some flowers are red’ one may first and foremost intend to state that ‘not all flowers are red’. So, isn’t Lacan’s (feminine) pastout a direct application of Brunschwig’s insightful distinction between, on one hand, the minimal ‘some’, privileged by Aristotle, allowing for the traditional square of opposition, and, on the other hand, a maximal ‘some’ objecting to universal propositions and saying ‘no’ to ‘all’, which sounds exactly like the de-universalizing proposition one may want to attribute to the feminine op-position to the universality of the phallic function? This is Guy Le Gaufey’s guiding thesis in his detailed analysis of the formulae, which will be discussed in the following paragraphs.

Le Gaufey’s logical approach to the formulae eventually results in this square of opposition:

FIGURE 5: Le Gaufey’s version of Lacan’s square

The reader immediately notices the differences with Lacan’s presentation of the formulae (see Figure 2). First of all, the formulae are identified according to the Aristotelian distinction between A, I, E and O; secondly, relations are added between the formulae, namely = (contradiction) and ↔ (equivalence), and thirdly, the formula ∀x Φx has switched places from the bottom right to the top right of the square. The advantage is obvious, for it provides – as the title of Le Gaufey’s book promises – a logical consistency to Lacan’s formulae. Although all three alterations deserve a critical examination – and, in my opinion, should be rejected – as they are based on Le Gaufey’s initial decision to read Lacan’s formulae as a square of opposition in which the particulars, I and O, are interpreted in a maximal way, the discussion will be limited to this.

For his decision, Le Gaufey finds inspiration in the aforementioned article by Brunschwig, who constructs the following diagram:

FIGURE 6: square of opposition with maximal I and O

Despite the differing symbolic notation – e.g. ‘Aa2B’ instead of “All A are B” – this square is indeed identical to the one including Lacan’s formulae, as Le Gaufey presents it. That we are dealing with a maximal particular is made clear by the subscript ‘2’ included in each formula, which is intended to differentiate it from the more common minimal interpretation and use of the square. The point not to overlook, however, is that Brunschwig presents this diagram in a conditional mode: “if one wants the couples a-o and e-i to remain contradictory, […]” As we have seen, in the Aristotelean square the universals A and E are contradictory with, respectively, the particulars I and O. If we interpret the latter in a maximal way, then these – I2 and O2 – are not only contradictory to their diagonally opposed universals – E2 and A2 – but also to the universals on the same side. This logically results in the equivalence of A2 and E2. At first sight, there are some problems with this reasoning. If we start at the bottom of the diagram, the equivalence of I2 and O2 is logically sound: if one defines ‘some’ as ‘some, but not all’, then ‘some A are B’ is equivalent with ‘some A are not B’, as both propositions mutually imply each other. If we now move to the relations between, respectively, A2 and I2, and E2 and O2, then it is less clear why one would call them contradictory. Contradictory means that both propositions can be neither true, nor false together – simply put, it is either the one or the other. Yet, an example makes us doubt this idea immediately: if I2 is ‘some trees are blue’, then A2 is ‘all trees are blue’, which makes it not too hard to imagine that both propositions can be false, i.e. when ‘no trees are blue’ (E2). If both propositions can be false, then they are not contradictory but contrary. And if A2 and I2 are contrary, and if – as we have seen – I2 and E2 are contradictory, then A2 and E2 cannot be equivalent, for they entertain
The first thing to observe here, is that if Lacan is logically articulating a difference it is definitely not a binary opposition; there is no opposition whatsoever between the ‘two’ sides, as both express (logically) one and the same opposition between a formula containing an existential (or particular) quantifier, \( \exists \), and a formula containing a universal one, \( \forall \). If there is any difference at stake, it can only be found in the interpretation of the quantifiers, in particular the \( \forall \)\(^{51}\).

On the left side, we find an unambiguous assertion according to which ‘all x are subjected to \( \Phi \)’ (\( \forall x \Phi(x) \)). This proposition is contradicted by the one above it, stating that ‘there is an x that is not subjected to \( \Phi \)’ (\( \exists x \). As already mentioned before, here we can locate the real of the symbolic universe whose subjects are all subjected to the phallic function. This universe, as Lacan argues, is only possible on the basis of the exclusion of the impossible, namely that there would be one who is not subject to the phallic law. The more technical point here, concerns the issue we encountered during our discussion of Frege, namely the constitution of a domain of x’s. How do we know what belongs to this domain, or, how do we single out x’s who serve as arguments satisfying function \( \Phi \)? In many commentaries of the left, so-called ‘masculine’ side of Lacan’s diagram, the x is identified as ‘the male (subject)’\(^{52}\). Even if one adds that this x could just as well be a biological woman – for sex is not to be reduced to natural characteristics – this seems to be missing the point of Lacan’s logical formalization of subjectivity and sexuality. The formulae on the left side express nothing more (or less) than that subjectivity – what one is dealing with in the psychoanalytic clinic – is defined by its being subjected to the phallic function. In order to constitute this domain of subjects, one needs to define it and here the impossible exception plays its role of providing the essential predicate, phallic castration, to be able to single out x’s belonging to the domain. In pseudo-mathematical language one could state that the exception draws the circle around the elements that belong to the class defined by \( \forall x \Phi(x) \). As Lacan puts it in one of the earlier seminars, any class presupposes a ‘classing’ (classement) and this does not happen through including elements to a pre-constituted class, but first of all by creating a class based on considering the absence of a certain trait as impossible\(^{53}\). This, moreover, makes evident the proximity and difference between different relations (contrariness and contradiction) to the equivalent \( \forall \) and \( \exists \) in brief, Brunschwig’s diagram looks like a forced and an altogether false construction of a ‘maximal’ square. Yet, this is too hasty a conclusion, for Brunschwig was careful enough to start the presentation of his ‘maximal’ square with a conditional if and to conclude it by pointing out the paradoxical nature of this square. Moreover, in the subsequent paragraph and in a clarifying footnote, makes clear that \( A_2 \) and \( E_2 \) are both defined as ‘all or all are not’, which simply means that \( A_2 \) and \( E_2 \) are not so much equivalent, but rather identical. The same holds for the particulars \( I_2 \) and \( O_2 \), for the maximal ‘some’ means in both cases ‘some are and some are not’.

This leads Brunschwig to conclude that, given the identity of \( A_2 \) and \( E_2 \), and of \( I_2 \) and \( O_2 \), the maximal square is not a square of opposition but “a segment of opposition” involving two instead of four propositions.\(^{48}\)

When extension and intension do not coincide
Brunschwig’s remark about the ‘maximal’ square of opposition serves as a good starting point for this concluding section on Lacan’s formulae of sexuality. For, indeed, many commentators mention the fact that from a classical logical point of view Lacan’s four formulae can easily be reduced to two.\(^{49}\)

 FIGURE 2: Lacan’s formulae of sexuality

In this diagram, \( \forall x \Phi(x) \) and \( \exists x \) are equivalent, as are \( \Phi(x) \) and \( \exists x \). Stating that ‘all x are subjected to \( \Phi \)’ (\( \forall x \Phi(x) \)) means exactly the same thing as stating that ‘there is no x which is not subjected to \( \Phi \)’ (\( \exists \exists x \). As already mentioned before, here we can locate the real of the symbolic universe whose subjects are all subjected to the phallic function. This universe, as Lacan argues, is only possible on the basis of the exclusion of the impossible, namely that there would be one who is not subject to the phallic law. The more technical point here, concerns the issue we encountered during our discussion of Frege, namely the constitution of a domain of x’s. How do we know what belongs to this domain, or, how do we single out x’s who serve as arguments satisfying function \( \Phi \)? In many commentaries of the left, so-called ‘masculine’ side of Lacan’s diagram, the x is identified as ‘the male (subject)’\(^{52}\). Even if one adds that this x could just as well be a biological woman – for sex is not to be reduced to natural characteristics – this seems to be missing the point of Lacan’s logical formalization of subjectivity and sexuality. The formulae on the left side express nothing more (or less) than that subjectivity – what one is dealing with in the psychoanalytic clinic – is defined by its being subjected to the phallic function. In order to constitute this domain of subjects, one needs to define it and here the impossible exception plays its role of providing the essential predicate, phallic castration, to be able to single out x’s belonging to the domain. In pseudo-mathematical language one could state that the exception draws the circle around the elements that belong to the class defined by \( \forall x \Phi(x) \). As Lacan puts it in one of the earlier seminars, any class presupposes a ‘classing’ (classement) and this does not happen through including elements to a pre-constituted class, but first of all by creating a class based on considering the absence of a certain trait as impossible.\(^{53}\)

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\[^{48}\text{Ibid., 8.}\]

\[^{49}\text{Yet, to my knowledge, none of them considers this awkward fact to be the key to 'unlock' the formulae. The apparent equivalence between the formulae is not a difficult yet secondary characteristic of Lacan's formulae (that one can ignore or should explain away), their equivalence is precisely the point of Lacan's logical argument.}\]

\[^{50}\text{Lacan puts the negation stroke above the quantifiers \( \forall \) and \( \exists \) and the function \( \Phi \). As we will see, this is done for a specific reason.}\]

\[^{51}\text{That is another reason not to follow Le Gauffe's presentation of the square: a logical square is based on the difference between two sides, whereas Lacan's 'square' may be not a square at all, but rather a one-sided 'universe' within which two formulae are repeated, with a small difference. This difference is indicated by the vertical line separating the two pairs of formulas, equivalent along a diagonal axis. The vertical line, therefore, should not be considered as a clear-cut division into two, but rather as the virtual line separating inside from outside on a Moebius strip.}\]

\[^{52}\text{See, e.g., Fink 1995, 104-125.}\]

logic and psychoanalysis. Both deal with the subject (the argument 'x' in assertions); and what this subject is, its essence, as Lacan puts it, "situates itself essentially in logic. It is a pure statement [énoncé] of discourse." The difference, however, is that logicians usually take the domain (or the universe) to which their propositions apply for granted, and if, as Lacan argues, the constitution of a domain requires an impossible proposition, then this impossibility, this 'impassé' is – for sound reasons – left out of the discussion. The neurotic subject is in that sense not a logical subject, for it entertains a complex relation – Oedipal, Freudians would argue – to this excluded impossibility or the homoinsou.

Moving to the right side, we have to address the difficulty that its two formulae are seemingly equivalent to the ones discussed above. This impression, however, is immediately contradicted by the divergent translations of one of Lacan's crucial notions pertaining to the right, 'feminine' side of the diagram. This notion is pastout, and one can find at least three different translations in English of it: not-whole, not-all and not-atany.

As the latter is a mere suggestion, the more interesting alternatives are not-whole and not-all, especially because they appear in officially translated volumes of Lacan's Seminar. The first option, not-whole, is the one chosen by Bruce Fink in his translation of Encore; the second one, of more recent date, is Adrian Price's choice, in his translation of ... or Worse, for 'not-all' as the English equivalent for pastout. In a translator's footnote, Price qualifies Fink's argument in support of his 'not-whole' "as flimsy as it is unprecedented". There is indeed no immediate reason to render Lacan's pastout into 'not-whole', for it seems to be a mere, although oddly written, negation of the classical, Aristotelian 'all'. Yet, without going into the details of Fink's argument, it is also clear that Lacan does not conceive of his pastout as a negation of 'all', i.e. a 'not every'. If one reads pastout as 'not every', one can only interpret $\exists x$ as 'not every $x$ is subjected to the phallic function', which is equivalent with the aforementioned $\exists x$ and turns $\exists x$ into a mere redundancy. Therefore, one can argue in favour of Fink's choice to avoid the quantitative 'not-all' (or 'not-every'), although the major disadvantage of 'not-whole' is its tempting suggestion to interpret the proposition as 'x is not-wholly subjected to the phallic function', leading to (or inspired by) the ubiquitous commentaries on Lacan's feminine as a subject that is not strictly castrated, or as a subject including a part that subtracts itself from castration.

The obvious retort to this sort of reading is that one does not need logic to state, let alone argue, that men and women, with respect to the phallic function, are different. This may be a clinical observation, but this also amounts to considering sexual difference as a fact. While, as we have seen, sexual difference may be empirical or clinically observed, its 'reality' can only be approached via logic. In brief, if we follow Lacan's torturous ways through the complexities of classical and modern logic, one should provide logical arguments for the basic intuition that any sexuality is hetero-sexuality, that is dividing the asexual unity of the domain, to which all subjects belong, into two.

The logical argument for the real of sexual difference is indeed related to Lacan's introduction of the pastout, one of two 'impasses' mentioned above. Again, it is fruitful not to limit one's reading to Encore, but to also pay attention to the first more or less sustained discussion of pas-tout. This occurs in the last session of Seminar IX, L'identification. There, Lacan makes use of the well-known saying 'all that glitters is not gold' (tout ce qui brille n'est pas or). This saying is a rather odd one, for one should not take its formulation literally to mean that all that glitters is not gold, but rather as not all that glitters is gold. Lacan mentions that he is not the first to discuss the phrase, but keeps his audience ignorant about who else may have shed some light on this negation of a universal statement. There is no conclusive evidence for this, but one can guess that Lacan is referring to Robert Blanché's article on negation and opposition, for in that text it is argued that there are two ways to the negate the affirmative universal, namely to deny either universally, or universality.

The author remarks that it is difficult to render this difference into French and therefore expresses it in Latin, omnis non and non omnis. He explains this in the following way: "We have recourse to Latin in order to avoid the equivoces of French, which usually does not put a negation in front of a complete proposition [that is non omnis]. Thus, Tout ce qui brille n'est pas d'or appears as a negative universal [...] although the meaning is obviously

54 Lacan 2006c [1971], 109. Here we touch upon a topic also present in Lacan's later seminars, that is the difference between the said (dit) and the saying (dire). One can relate this to the issue of universality and its constitutive exception, in the sense that what is said (the universal) relies on a primordial saying (the exception) that provides a symbolic being (an essence) to any x (or the subject) belonging to the domain.


60 One could relate this to the O-corner's difficulty discussed in footnote 9: it would be easier to state that 'some glittering things are not gold', but if one wants to emphasize that it is definitely not all of them, there is no immediate word available, hence the addition of a negation in front a proposition that starts with 'all' (and which may explain this negation's capacity to move from its proper place, in front, to a less logically precise position in the sentence).

61 Blanché 1957, 190-191.
The Logic of Lacan’s Not-All

In most cases Lacan seems to reject the distinction having it establishes between subject and predicate a relation of which Lacan discusses the Aristotelian ‘all’ and the question whether be related to the paragraphs right before them, one of many passages in of gold. The saying and the questions it elicits on Lacan’s behalf, should this be done by a constitutive exception, allowing for the universality of ‘All x is subjected to the phallic function’. This means that the domain consists of x’s that satisfy the function Φ. Therefore, in this case, the extension and intension (the ‘compréhension’ or ‘meaning’ referred to above) of ∀x Φx mutually imply each other. Its extension is all the x’s belonging to the domain, its intension concerns the definition of x, what this x is, or in less logical terms, what allows us to recognize such an x as an x belonging to the domain – which is, in this case, ‘being subjected to the phallic function’. There is no other trait, characteristic, attribute or predicate (intension) that allows us to count an x as an x belonging to the domain (extension). This is, of course, in conformity with the basic axiom of Lacanian psychoanalysis: a subject is a subject of castration, and there is no other definition available or any other predicate that would give an x ‘access’ to the domain.

This distinction was already known to Aristotle, for he differentiates between three different meanings of the universal ‘all’: 1. kātā pantos, 2. kath’auto and 3. kathōloû. The first use is identical to what is currently named extension; the second one refers to intension – the predicate expresses something essential about, a ‘defining’ aspect of the subject – and the third use expresses what one calls a ‘commensurate universal’, relating a predicate to all subjects (extension), in an essential way (intension), establishing co-extensivity between the subject and its predicate. The latter criterion simply means what we have been describing above regarding the Lacanian subject: all subjects are castrated and all what is castrated is a subject. One could also describe it as the case where subject and predicate are indistinguishable, not...
even allowing for questions about the possible difference between the extension and intension of $\forall x \Phi x$.

This, however, is precisely what the ‘feminine’ side of Lacan’s diagram does: seemingly, it is a mere repetition of the ‘masculine’ side, yet it drives a wedge between the extension and the intension of $\forall x \Phi x$.

This reading is corroborated by several passages in which Lacan refers to the difference between pan(tès) and holon, or, in Latin, between omnis and totus, or, in modern logical language, between extension and intension.\(^{68}\) The pastout read in a classical way, may lead one to conclude that it is equivalent with $\exists x $.\(^{68}\) Yet, as we have seen, the pastout is not a straightforward denial of affirmative universality – it definitely does not mean $\exists x$ or $\forall x$ – it means that ‘all x are subjected to the phallic function’ extensionally but not in any intensional way.\(^{69}\) The quantifier pastout\(^{20}\) – invented by Lacan in response to Blanché and other logicians dealing with the problem of ex- and intensionality, and rendered as a negation not in front, but above the $\forall$ (cf.), pastout in one word and not pas tout or pas tous – indicates that all x are subjected to the phallic function (extension), but do not constitute an ‘all’, that is a class or a domain defined by this particular function (intension). All the elements of the set defined by $\Phi x$ do not constitute a class. In a way analogous to the example discussed above, ‘not all that glitters is gold’, the pastout puts the function (gold or, in this case, phallus) into question: if the phallus determines subjectivity – turns all x into a subject – then the not-all introduces doubt whether the phallus suffices to do so. This does not mean that one can use any other function than the phallic one to identify the Lacanian subject or, conversely, that there are subjects who are not subjects of the phallic function. The mere difference between $\forall x \Phi x$ and $\Phi x$ resides therein, that the latter proposition allows for subjects of the phallic function who are not essentially characterized as such. If one were to identify those as ‘women’, one can state that they do have an existence, yet no essence as subjects of the phallic function. The latter idea is expressed in many different ways by Lacan, from the famous La femme n’existe pas – there is no the woman, for this would imply the possession of an essential trait\(^{71}\) – to more oblique formulations such as for women the phallus does not function as cause (i.e., it does not determine them as subjects), they are not linked to castration essentially\(^2\) or the pastout is “an all outside of ‘universe’” (un tout d’hors univers), that is not the negation of ‘all’, but an ‘all’ that negates its own capacity to constitute a universe (or domain).\(^{73}\)

Conclusion

Whether there is a non-binary difference between men and women, was the guiding question in our step-by-step reconstruction of the formulae. Now, we can locate this difference in the pastout, which is “an objection to the universal”.\(^{74}\) This universal, $\Phi$, concerns both men and women as subject; they may relate differently to it, but that does not alter their status as subjects of the phallic function. The difference, therefore, is not a mere negation of the function $\Phi$, for, on the contrary, this function has as its argument all x. The difference or ‘logical impasse’ concerns the constitution of the domain to which all x belong. On the ‘masculine’ side of the formulae, there is a short-circuit between what the subject is and what it has qua discriminating feature, i.e. phallic castration. This side is not contradicted by the other, ‘feminine’ one, but simply opened up by a formula that says ‘yes, all are subjected the phallic function, but no, that is not what we are.’ Or, put differently, men’s existence coincides with their essence (= symbolic being), whereas women, as the other sex, suggest the possibility of an existence without an essence.

How is this related then to the absence of sexual relation? Paradoxically enough, sexuality is the field where the partner appears as the a-sexual object of desire. A sexual relation would imply that it is based upon or produces a common ground between two different positions. Yet, as we have seen there is no difference, only an obstacle dissolving any (shared) universality. Here we encounter the two dimensions of the object a. On the one hand, it is the ir-rational object, the part without a whole, the thing that exists without having a being (i.e. a symbolic essence), and as such the incarnation of the obstacle to universality of the symbolic. On the other hand, it is a phantasmatic object, creating the illusion of a possible completeness. Sexuality may be not all, the object a occupies the place of and veils the notall in the phantasm.\(^{75}\)

\(^{67}\) Lacan 2018 [1971-1972], 35.

\(^{72}\) Lacan 2001, 466. This is ‘confirmed’ by the other ‘feminine’ formula, which is equivalent with $\forall x \Phi x$, but also denies that there is a constitutive exception to the universality of $\forall x \Phi x$. All women are subjected to the phallic function, but there is no universe (intensionally speaking) of ‘woman’.

\(^{73}\) Lacan 1967-1968, 28 February and 20 March 1968. This allows for an interpretation of Lacan’s seemingly offhand allusion to Pascal’s well-known pensive: “[...] anyone trying to act the angel acts the beast.” Lacan 1998 [1972-1973], 20; Pascal 1966, 242; see also 60. In sexuality the other appears as an a, with whom one performs an ‘angelic’, sexless act, only to (re-)produce the mute (désé) signifier that governs the subject as a subject of the signer. This universal coincidence of angelism and animality, however, is supplemented by a contingent not-all.